Tutorial 2 for MATH 2020A (2024 Fall)

1. Let $\Omega \subset \mathbb{R}^3$ be the solid which is the wedge cut from the first octant by the cylinder $z = 12 - 3y^2$ and the plane $x + y = 2$. Sketch Ω and find its volume.

Solution: 20

2. Consider the following iterated integral

$$\int_0^3 \int_0^{2-\frac{2x}{3}} \left(1 - \frac{1}{3}x - \frac{1}{2}y\right) \mathrm{d}y \, \mathrm{d}x.$$

(a)Calculate its value.

(b)Sketch the solid $\Omega \subset \mathbb{R}^3$ whose volume is given by this double integral.

(c)Use the volume formula of tetrahedron $V = \frac{1}{3} \times$ **bottom area** $\times$ **height** to verify your answer in (a).

Solution: (a) 1

3. Consider the following sum of integrals

$$\int_{-1}^0 \int_{-2x}^{1-x} \mathrm{d}y \, \mathrm{d}x + \int_0^2 \int_{-\frac{x}{2}}^{1-x} \mathrm{d}y \, \mathrm{d}x.$$

(a)Calculate its value.

(b)Sketch the area $R \subset \mathbb{R}^2$ whose area is given by this sum of integrals.

(c)Use the area formula of triangle to verify your answer.

Solution: (a)$\frac{3}{2}$

4. If $f(x, y) = \dfrac{10^4 e^y}{1 + \frac{|x|}{2}}$ represents the "population density" of a certain bacterium on the $xy-$plane, where x and y are measured in centimeters, find the total population of bacteria within the rectangle $-5 \leq x \leq 5$ and $-2 \leq y \leq 0$.

Solution: $4 \times 10^4 \times (1 - e^{-2}) \times \ln \frac{7}{2}$

5. Choose a suitable coordinate to evaluate the following integral

$$\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}}\,\mathrm{d}y\,\mathrm{d}x.$$

Solution: $\pi(1-\ln 2)$